Exercise - monetary policy rules

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1 Exercise

1. Suppose that the economy is described by the simple monetary model taught in class. Suppose that the shock process for productivity is given by:

$$a_t = \rho_a a_{t-1} + \varepsilon_t \tag{1}$$

Where $E_t(\varepsilon_{t+1}) = 0$ and that the monetary policy rule is given by:

$$i_t = \rho + \phi_\pi \pi_t$$

Find the equilibrium level of inflation.

2 Answer

Supposing that $\phi_\pi > 1$, we have the solution:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} [E_t \hat{r}_{t+k}]$$

Noting that:

$$\hat{r_t} = r_t - \rho$$

And:

$$r_t = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1}$$

We can rewrite the equation as:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} [\sigma \psi_{ya} E_t \Delta a_{k+1}]$$

Now, we need to determine:

$$E_t \Delta a_{k+1}$$

Note that:

$$a_{t+1} = \rho_a a_t + \varepsilon_{t+1}$$

$$\Delta a_{t+1} = a_{t+1} - a_t$$

$$\Delta a_{t+1} = \rho_a a_t + \varepsilon_{t+1} - a_t$$

$$\Delta a_{t+1} = (\rho_a - 1)a_t + \varepsilon_{t+1}$$

Taking expectations:

$$E_t(\Delta a_{t+1}) = (\rho_a - 1)a_t$$

Do the same for t+2:

$$\Delta a_{t+2} = (\rho_a - 1)a_{t+1} + \varepsilon_{t+2}$$

Go back to t:

$$\Delta a_{t+2} = \rho_a(\rho_a - 1)a_t + \varepsilon_{t+2} + \rho_a\varepsilon_{t+1}$$

Take expectations again:

$$E_t(\Delta a_{t+2}) = \rho_a(\rho_a - 1)a_t$$

At n we will have:

$$E_t(\Delta a_{t+n}) = \rho_a^n(\rho_a - 1)a_t$$

So we can write the equation 2 as:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)}[\rho_a^k \sigma\psi_{ya}(\rho_a - 1)a_t]$$

$$\pi_t = \sigma\psi_{ya}(\rho_a - 1)a_t \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)}[\rho_a^k]$$

To obtain the solution to the geometric progression:

$$\sum_{k=0}^{\infty} \phi_\pi^{-(k+1)}[\rho_a^k]$$

First write it as:

$$\frac{1}{\phi_\pi} \sum_{k=0}^{\infty} (\frac{\rho_a}{\phi_\pi})^k$$

Which simplifies to:

$$\frac{1}{\phi_\pi} \frac{1}{1 - \frac{\rho_a}{\phi_\pi}}$$

and finally to:

$$\frac{1}{\phi_\pi - \rho_a}$$

So the solution for inflation is:

$$\pi_t = \frac{\sigma\psi_{ya}(\rho_a - 1)a_t}{\phi_\pi - \rho_a}$$